Calculus Warm Up #4-1

Evaluate. No Calculator.

1. $\displaystyle\int_0^1 \frac{x}{x^2 - 4}\,dx$

2. $\displaystyle\int_0^{\pi/6} (\cos x)e^{\sin x}\,dx$

3. Find the total area of the region between the curve and the x-axis. No calculator.

$y = 3x^2 - 3$ on $[-2, 2]$

HW Questions: p. 278

In Exercises 29–38, evaluate the indefinite integral by the method shown in Example 5.

29. $\displaystyle\int x\sqrt{x+2}\,dx \longrightarrow = \frac{2(x+2)^{5/2}}{5} - \frac{4(x+2)^{3/2}}{3} + C$

31. $\displaystyle\int x^2\sqrt{1-x}\,dx$

33. $\displaystyle\frac{1}{2}\int \frac{2(x^2-1)}{\sqrt{2x-1}}\,dx$

$u = 2x - 1 \quad du = 2dx$

$x^2 = \left(\frac{u+1}{2}\right)^2 = \frac{1}{4}(u^2 + 2u + 1)$

$$\frac{1}{2}\int \frac{\left(\frac{u^2+2u+1}{4} - \frac{4}{4}\right)du}{\sqrt{u}}$$

$$\frac{1}{8}\int (u^2 + 2u - 3)u^{-1/2}\,du$$

35. $\int \frac{-x}{(x+1)-\sqrt{x+1}}\,dx$ 37.

Let $u = x+1 \quad du = dx$

$x = u - 1$

$-\int \frac{(u-1)}{(u-\sqrt{u})}\,dx \cdot \frac{u+\sqrt{u}}{(u+\sqrt{u})}$

$-\int \frac{(u-1)(u+\sqrt{u})}{u^2-u}\,du \longrightarrow u(u-1)$

$-\int \frac{u+\sqrt{u}}{u}\,du$

In Exercises 39–50, evaluate the definite integral.

39. $\int_{-1}^{1} x(x^2+1)^3\,dx$

41. $\frac{1}{2}\int_0^4 \frac{2}{\sqrt{2x+1}}\,dx$

43. $\int_1^9 \frac{1}{\sqrt{x}(1+\sqrt{x})^2}\,dx$

Let $u = 2x+1 \quad du = 2\,dx$

for $x = 0 \rightarrow u = 1$

$x = 4 \rightarrow u = 9$

$\frac{1}{2}\int u^{-1/2}\,du$

In Exercises 39–50, evaluate the definite integral.

45. $-\int_1^2 -(x-1)\sqrt{2-x}\,dx$ → let $u = 2-x$ $du = -dx$
$x = 2-u$ for $x=1 \to u=1$
$x=2$ $u=0$

47. $\int_3^7 x\sqrt{x-3}\,dx$

49. $\int_0^7 x\sqrt[3]{x+1}\,dx$

$$-\int_1^0 (2-u-1)\sqrt{u}\,du$$

$$-\int_1^0 (1-u)u^{1/2}\,du$$

$$-\int_1^0 \left(u^{1/2}-u^{3/2}\right)du$$

$$-\left[\frac{2u^{3/2}}{3}-\frac{2u^{5/2}}{5}\right]_1^0$$

$$-\left[0 - \right.$$

In Exercises 39–50, evaluate the definite integral.

45. $\int_1^2 (x-1)\sqrt{2-x}\,dx$

47. $\int_3^7 x\sqrt{x-3}\,dx$ → let $u = x-3$ $du = dx$
$x = u+3$ $x=3 \to u=0$
$x=7 \to u=4$

49. $\int_0^7 x\sqrt[3]{x+1}\,dx$ $u = x+1$ $du = dx$
$x = u-1$ $x=0 \to u=1$
7 $u=8$

$$\int_1^8 (u-1)u^{1/3}\,du$$

$$\int_1^8 \left(u^{4/3}-u^{1/3}\right)du$$

51. Use the fact that

$$\int_0^2 x^2\,dx = \frac{8}{3}$$

to evaluate the following definite integrals without using the Fundamental Theorem of Calculus.

(a) $\int_{-2}^{0} x^2\,dx$ (b) $\int_{-2}^{2} x^2\,dx$

(c) $\int_0^2 -x^2\,dx$ (d) $\int_{-2}^{0} 3x^2\,dx$

5.6 Trapezoid Rule

Simpson's Rule

Why approximate the area using right or left endpoints, midpoints or trapezoids?

Sometimes we can't find the antiderivative using the Fundamental Theorem!

For example, there are no functions whose derivatives are:

$$\sqrt[3]{x}\sqrt{1-x} \qquad \sqrt{1-x^3}$$

The trapezoid rule gives us a pretty close approximation, but has its limitations.

(a straight top)

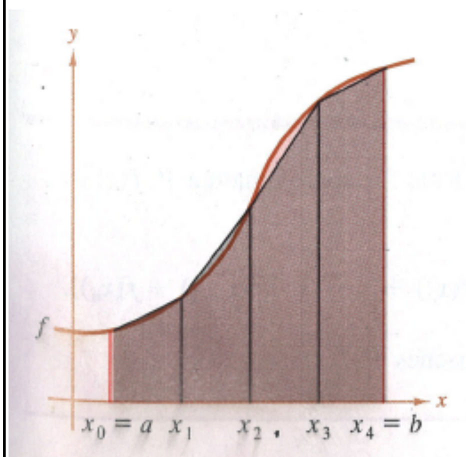


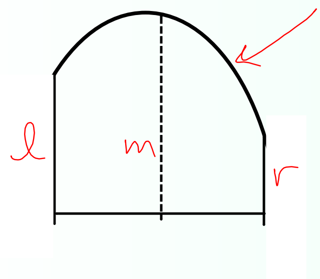
FIGURE 5.33

What if there was a formula for area of a geometric figure with straight sides and bottom, but a curved top?

The ancient Greeks had that figured out!

Area under a parabola:

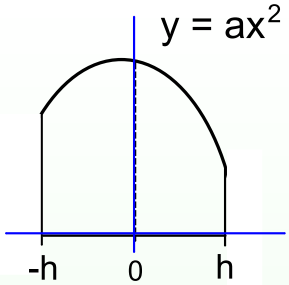
$y = ax^2 + bx + c$



$A = \frac{h}{3}(\ell + 4m + r)$

h = half the base
ℓ = height of left side
m = height @ midpoint
r = height of right side

$y = ax^2 + bx + c$



$A = \frac{h}{3}(\ell + 4m + r)$

Proof

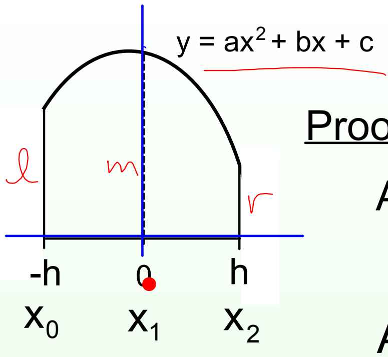
$$A = \int_{-h}^{h} (ax^2 + bx + c)\, dx$$

$$A = \frac{ax^3}{3} + \frac{bx^2}{2} + cx \Big]_{-h}^{h}$$

$$= \frac{ah^3}{3} + \cancel{\frac{bh^2}{2}} + ch - \left(\frac{a(-h)^3}{3} + \cancel{\frac{b(-h)^2}{2}} + c(-h)\right)$$

$$= \frac{2ah^3}{3} + \frac{2ch \cdot 3}{3}$$

$$= \frac{h}{3}(2ah^2 + 6c)$$



$$A = \frac{h}{3}(\ell + 4m + r)$$

Proof

$$A = \int_{-h}^{h} (ax^2 + bx + c)\, dx$$

$$A = \frac{h}{3}(2ah^2 + 6c)$$

$\ell = f(x_0)$

$m = f(x_1)$

$r = f(x_2)$

$$(\ell + 4m + r) \stackrel{?}{=} (2ah^2 + 6c)$$

$$f(-h) + 4f(0) + f(h)$$

$$a(-h)^2 + \cancel{b(-h)} + c + 4c + ah^2 + \cancel{bh} + c$$

$$ah^2$$

$$2ah^2 + 6c$$

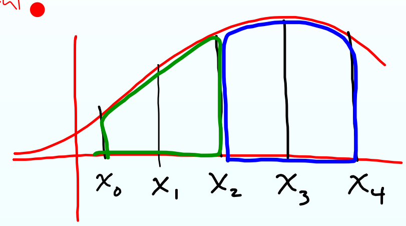
✓

Using Simpson's Rule:

n= even # of subintervals

every 2 will make up a parabola region with a mid-height

Let n = 4

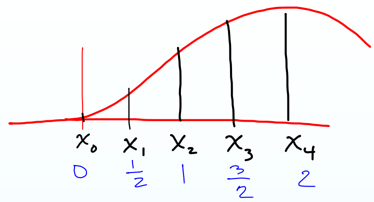


$$A = \frac{h}{3}(\ell + 4m + r)$$

$$A = \frac{h}{3}\left[f(x_0) + 4f(x_1) + \underbrace{f(x_2) + f(x_2)}_{2f(x_2)} + 4f(x_3) + f(x_4)\right]$$

Let n = 4

$$\int_0^2 5x^4 dx = \boxed{32}$$



$f(x) = 5x^4$

$$h = \frac{b-a}{n} \qquad h = \frac{2-0}{4} = \frac{1}{2}$$

$$\frac{h}{3} = \frac{1}{6}$$

$$A = \frac{1}{6}\left[f(0) + 4f\left(\tfrac{1}{2}\right) + 2f(1) + 4f\left(\tfrac{3}{2}\right) + f(2)\right]$$

$$= \frac{1}{6}\left[0 + \cancel{4}\left(\frac{5}{\cancel{16}_{4}}\right) + 2(5) + 4\left(\frac{405}{16}\right) + 80\right]$$

$$= \frac{1}{6}\left[\frac{5}{4} + 90 + \frac{405}{4}\right]$$

$$\approx 32.083$$

THEOREM 5.20
SIMPSON'S RULE (n is even)

Let f be continuous on $[a, b]$. Simpson's Rule for approximating $\int_a^b f(x)\,dx$ is given by

$$\int_a^b f(x)\,dx \approx \frac{b-a}{3n}[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \cdots + 4f(x_{n-1}) + f(x_n)].$$

Moreover, as $n \to \infty$, the right-hand side approaches $\int_a^b f(x)\,dx$.

For the HW, we have already done all the trapeziod approximations. Use your past work to compare those outcomes to what you get with Simpson's Rule.

HW: p. 287 # 1 - 11 odd

(just Simpson's rule)